

This text, which is based on a series of lectures on the LISP programming language and on REDUCE, appears to target an audience of persons who have acquired a REDUCE system but are (a) completely unfamiliar with algorithms for symbolic computation, and are (b) interested in seeing some applications.

Chapter 1 is a very brief (17-page) introduction to computer algebra systems' general capabilities. See Buchberger et al. [1] for a collection of more accurate reports and bibliographic information.

Chapter 2 (88 pages) describes "Standard LISP". Since REDUCE is written in this language (actually, Portable Standard LISP), it is necessary to know LISP to gain an in-depth understanding of the internal operation of REDUCE. Yet persons learning LISP for the first time should certainly seek an alternative to this treatment. The authors dwell on those features that should be avoided in writing programs in LISP, yet ignore important concepts such as data abstraction.

Since Brackx and Constales go to the effort of presenting LISP, one might reasonably expect to learn how the system REDUCE is written, preferably described in layers of abstraction covering up the gritty implementation details. Such a description could demonstrate to the novice how easy it is to (say) differentiate expressions. But this is entirely missing.

Chapter 3 is a summary of the REDUCE user manual.

Chapter 4 is a collection of brief programming examples, and Chapter 5 is a 50-page description of a set of programs dealing with Euclidean geometry (for example, given a certain description of a figure, rotate it). The 9-entry bibliography and the 6-page index do not strengthen the book.

I found the title of this book misleading since there is virtually no description of how REDUCE performs any of its computer algebra, nor is there any indication of what is easy or difficult to automate in mathematics generally, or why. The promise of an "introduction to computer-aided pure mathematics" is unfortunately not fulfilled.

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1. B. Buchberger, G. E. Collins, and R. Loos (eds.), *Computer algebra: Symbolic and algebraic computation*, 2nd ed., Springer-Verlag, Wien and New York, 1983.

12[13Pxx, 14Qxx, 20B40, 20C40, 68Q40].—G. M. PIACENTINI CATTANEO & E. STRICKLAND (Editors), *Topics in Computational Algebra*, Dordrecht: Kluwer Academic Publishers, 1990, iii + 261 pp., 24½ cm. Price \$99.00/Dfl.160.00.

Computational algebra is a quickly expanding area, in which good books are still rare. Hopeful expectation put me in a forgiving mood when I started to review this book.

Therefore, the graphic design of *Topics in Computational Algebra* will not be ascribed to the lack of taste of anybody in particular; the amateuristic artwork on the cover (showing a personal computer that must have looked old-fashioned at least a decade ago) starkly contrasts with the choice of expensive acid-free paper and a plasticized hard cover by the publisher. Likewise, I am willing to

assume that the use of at least a dozen different fonts and a bottom margin that allows you to prove just about every theorem in it are consequences of the fact that this book is a reprint of the numbers 1 & 2 of Vol. 21 of *Acta Applicandae Mathematicae*.

We should be grateful to the editors Piacentini Cattaneo and Strickland for putting so much energy in first organizing a Semester on Computational Algebra in Rome, and next squeezing enough papers out of ten invited speakers to fill this book with. Surely, after coming up with an appropriate title, they could impossibly have found the time to write a proper Introduction to the book, for instance to substantiate their view (expressed in the half-page Foreword) that “the Semester largely succeeded in reaching its intended objectives”, which were “to give an update on interesting techniques based on algorithms in different branches of algebra, with the idea of emphasizing the computational aspects of each branch”. Mind you, they do not dare to state that the same objective applies to this book. (In fact the task of finding a common denominator to the papers is a hard one and reminded me of the impossible high school exams for which one had to discover a ‘theme’ in some experimental novel.) Hence, one can hardly blame contributors for not submitting the material that best suited these objectives. At least in one case such material did exist, even though it is not presented here, but luckily we are reassured by Neubüser that one can recover most of it by tracing back the literature in the bibliography to his joint paper with Celler and Wright, *Some remarks on the computation of complements and normalizers in soluble groups*. This paper, in which new methods and implementations in the Aachen GAP system for the calculation of classes of complements of a normal subgroup and of normalizers of a subgroup in a soluble group are described, and compared with existing implementations in the CAYLEY language, could certainly make a nice article in a journal on algorithms and computations in algebra. But in this book I would have preferred to see the survey that Neubüser apparently delivered in three lectures under the promising title *Computing in Soluble Groups* in another form than a list of references.

The next paper (*Methods for computing in algebraic geometry and commutative algebra* by Stillman) does not even give us a complete list of references (“Find Reference” is listed as item [12] instead)—however, we will not blame the author for this, nor for photocopying three pages of computer listings with his remarks scribbled in the margin, making a fresh copy of the book look secondhand already. But at least this contribution does not need a bibliography to make it an ‘interesting and suitable update on interesting techniques’, in this case all based on Gröbner basis calculations, ‘with the idea of emphasizing the computational aspect’. On his way, the author explains what the main goals are in computational algebraic geometry and commutative algebra, and how these are achieved in the Macaulay system (of Stillman and Eisenbud). As an example the algorithm for calculating the radical of an ideal in a polynomial ring is described. In *Computing with characters of finite groups* by H. Pahlings, the application of another important technique, the ‘LLL’ method of finding short vectors in lattices, to the calculation of ordinary characters of finite groups is very clearly outlined. N. White goes out of his way to explain the importance of Cayley algebras and Cayley factorization of bracket polynomials in the algebraic interpretation of geometric statements, in his *Cayley factorization and a straightening algorithm*.

To offset these three relevant papers, there are two or three whose inclusion in the book I found hard to justify, no matter how lenient the mood. Formanek's *The Nagata-Higman theorem* describes the history of the theorem which states that if in an algebra over a field of characteristic zero every n th power vanishes, then for some $N = d(n)$ every product of N elements vanishes. This theorem is 35 years old and with respect to the only aspect that with my good will I could call computational (that of determining $d(n)$) no progress is reported in 15 years: but "the reader may feel that, given the inequality $n(n+1)/2 \leq d(n) \leq n^2$ the exact value of $d(n)$ no longer is of sufficient interest", as Formanek puts it. Perhaps afraid that the same qualification would apply to his contribution, he concludes with a vaguely related (but at least recent) result concerning Engel conditions. Although obtaining multiplicities of representations in a decomposition of an irreducible representation of a group with respect to a subgroup could have a computational flavor, Kac and Wakimoto succeed very well in avoiding calculations and mention of the word algorithm in the description of the particular case they are interested in (*Branching functions for winding subalgebras and tensor products*). Especially since this is the very first paper of the volume it seems a trifle too specialistic: in the smallest of all fonts used in the book, the Introduction and the Preliminaries take up almost half of the 37 pages of the paper. On the other hand, *Supersymmetric bracket algebra and invariant theory* could have done with an introduction, for instance to highlight the computational and the new aspects of the subject to the reader, but the authors (Huang, Rota and Stein) may have considered that quite rightly to be a task for the editors.

Buchsbaum, in *Aspects on characteristic-free representation theory of GL_n , and some application to intertwining numbers*, explicitly "tried to choose topics that indicate a strong bridge between representation theory and computational algebra": he illustrates the use of Schur complexes in the calculation of certain intertwining numbers. In *The expansion of various products of Schur functions*, Remmel outlines recent developments on efficient combinatorial algorithms for computing various products of symmetric functions, and the application to the representation theory of the symmetric group. Finally, Drensky purports to give a quantitative description of the polynomial identities of algebras of 2×2 matrices over a field of characteristic zero, without using invariant theory.

Perhaps you think this is not a proper book review; well, I am of the opinion that this is not a proper book: some collections of papers, however well-intended, put (and bound) together, still don't make up more than a journal. Hopefully you agree with me, in a final flattering moment of forgiving, that this is just as good a review as any journal deserves.

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